

Blackbody friction force on a relativistic small neutral particle

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The friction force acting on a small neutral particle during relativistic motion relative to the blackbody radiation is calculated in the framework of fluctuation electrodynamics. It is shown that the particle acceleration is determined by the friction force in the particle rest reference frame (K' -frame) which in general is not equal to the friction force in the frame of the blackbody radiation (K -frame). The difference between the friction forces in the different frames is connected with the change of the rest mass of a particle due to the absorption and emission of radiation. The friction force in the K' -frame is determined only by the interaction of a particle with the blackbody radiation. In the K -frame the interaction of a particle with its own thermal radiation also contributes to the friction force. For the steady state temperature of a particle the friction forces in the K' - and K -frames are equal. For an atom the blackbody friction is determined by the electronic linewidth broadening which is calculated considering the interaction of an atom with its own radiation. In the ultrarelativistic case ($1 - \beta \rightarrow 0$) for an atom the friction force diverges as $(1 - \beta)^{-3}$ and the (average) temperature of an atom $T_2 \approx (1 - \beta)^{-3/8} T_1$, where T_1 is the temperature of the blackbody radiation and $\beta = V/c$. Controversies in the theory of the blackbody friction are discussed.

I. INTRODUCTION

A remarkable example of the forces induced by fluctuations is the friction force acting on a particle at motion relative to the fluctuating electromagnetic field created by thermal and quantum fluctuations. At present the radiation friction is attracting a lot of attention due to the fact that it is one of the mechanisms of non-contact friction between bodies in the absence of direct contact¹. The non-contact friction determines the ultimate limit to which the friction force can be reduced and, consequently, also the force fluctuations. The force fluctuations (and hence friction) are important for ultrasensitive force registration. Perhaps the most exciting application of these ideas is associated with mechanical detection of nuclear spin resonance². For example, a single spin detection using magnetic resonance force microscopy³ (which was proposed to obtain images of biological objects, such as proteins, with atomic resolution) and for quantum computing⁴ would require reducing the fluctuating forces (and therefore friction) to unprecedented levels. In addition, the measurements of gravitation interactions at a small length scale⁵, and the future measurements of the Casimir forces⁶ and the behavior of micro- and nano-electromechanical devices may eventually be limited by non-contact friction effects.

Radiative friction has deep roots in the foundation of quantum mechanics. Friction force during the motion of a particle relative to the blackbody radiation, which is a limiting case of the fluctuating electromagnetic field, was studied by Einstein and Hopf⁷ and Einstein⁸ in the early days of quantum mechanics. The friction in this case can be explained by the direction-dependent Doppler effect. In the rest reference frame an atom absorbs blue-shifted counterpropagating blackbody photons, while emitting these photons in all directions which leads to the momentum transfer and friction. Blackbody radiation friction is connected with the interaction of atoms with the cosmic blackbody radiation^{9–11}. The friction induced by electromagnetic fluctuations was studied in the plate-plate^{1,12–16}, neutral particle-plate^{1,17–26}, and neutral particle-blackbody radiation^{1,11,16,19,22,27–30} configurations. However, the theory of radiative friction is still controversial. As an example, different authors have studied the friction force between a small neutral particle and blackbody radiation^{1,11,16,19,22,27–30}, using different approaches, and have obtained results which are in sharp contradiction to each other. In Refs.^{1,7,8,11,16} only interaction of a particle with blackbody radiation was taken into account, therefore the friction force depends only on the temperature of this radiation. On the other hand, in Refs.^{19,22,27} the interaction of a particle with its own thermal radiation was also taken into account, in this case the friction force depends on the temperatures of both a particle and the blackbody radiation. Controversies between different theories of blackbody radiation friction recently were discussed in Ref.²⁷.

In the present paper a general theory of the blackbody friction for a neutral particle at relativistic motion relative to the blackbody radiation which includes as limiting cases the previous theories from Refs.^{1,11,16,19,22,27–30} is developed. This general theory establishes a link between the different theories of the blackbody friction. In sharp contrast to the

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opinion of the authors of Ref.²⁷ the total friction forces acting on a particle in the frame of the blackbody radiation (K -frame) and in the rest frame of a particle (K' -frame) are not equal. This difference is due to the change of the rest mass of a particle as a result of the absorption and emission of radiation by a particle. Recently, Ref.²² used a fully covariant formulation of the blackbody friction. However, the physical origin of the difference of the friction force in the different frame and the condition for the steady state temperature of a particle were not established in that article.

II. THEORY

We introduce two reference frames, K and K' . K is the frame of blackbody radiation, and K' moves with velocity V along the x axis. In the K' frame a particle moves with velocity $v' \ll V$ along the x -axis. At the motion of a particle with acceleration the K' frame coincides with the particle rest reference frame only at $t = t_0$ when $v'(t_0) = 0$. However, for $v' \ll V$ the difference between the friction forces in the K' frame and the particle rest reference frame can be neglected thus in this paper the K' frame is denoted as the particle rest reference frame. The relation between the x components of the momentum in the different reference frames is given by

$$p_x = (p'_x + \beta E'/c)\gamma, \quad (1)$$

where $\beta = V/c$, $\gamma = 1/\sqrt{1-\beta^2}$, $E' = E_0/\sqrt{1-(v'/c)^2}$ is the total energy of a particle in the K' frame, and $E_0 = m_0c^2$ is the rest energy of a particle. The rest energy can change due to the absorption and emission of thermal radiation by a particle. The connection between forces in the K and K' frames follows from Eq. (1)

$$\frac{dp_x}{dt} = \frac{1}{1+(Vv'/c^2)} \left[\frac{dp'_x}{dt'} + V \frac{dm_0}{dt'} \frac{1}{\sqrt{1-(v'/c)^2}} + m_0 V \frac{d}{dt'} \left(\frac{1}{\sqrt{1-(v'/c)^2}} \right) \right]. \quad (2)$$

For $v' \ll V$ from (2) we get

$$F_x = F'_x + V \frac{dm_0}{dt'}, \quad (3)$$

where F_x and F'_x are the forces in the K and K' frames, respectively. The last term in Eq. (3) determines the rate of change of the momentum of a particle in the K frame due to the change of its rest mass as a result of the absorption and emission of radiation by a particle. Taking into account that at $v' \ll V$

$$\frac{dp_x}{dt} = \frac{d}{dt} \left(\frac{m_0 v}{\sqrt{1-(v/c)^2}} \right) = \frac{dm_0}{dt'} V + m_0 \gamma^3 \frac{dv}{dt}, \quad (4)$$

from Eq. (3) we get

$$m_0 \gamma^3 \frac{dv}{dt} = \frac{dp'_x}{dt'} = F'_x, \quad (5)$$

where v is the velocity of a particle in the K frame. From Eq. (5) it follows that acceleration in the K frame is determined by the friction force in the K' frame.

At uniform motion of a particle an external force f_x should be applied to it. At $V = \text{const}$ the equation of the motion of a particle is

$$\gamma V \frac{dm_0}{dt} = F_x + f_x. \quad (6)$$

If the force f_x does not change the rest mass of a particle, then its value is the same in the K and K' frames, i.e., $f_x = f'_x$. In this case

$$f_x = -F_x + \gamma V \frac{dm_0}{dt} = -F_x + \frac{dm_0}{dt'} V = -F'_x, \quad (7)$$

that is, for uniform motion of a particle, an external force that is equal but opposite in sign to the friction force in the K' frame should be applied to it.

In the K' frame the Lorentz force on the particle is determined by the expression^{18,31}

$$F'_x = \frac{\partial}{\partial x'} \langle \mathbf{d}'_e \cdot \mathbf{E}'^*(\mathbf{r}') \rangle_{\mathbf{r}'=\mathbf{r}'_0}, \quad (8)$$

where according to the fluctuation electrodynamics³²⁻³⁴ $\mathbf{d}'_e = \mathbf{d}_e^{f'} + \mathbf{d}_e^{in'}$, $\mathbf{E}' = \mathbf{E}^{f'} + \mathbf{E}^{in'}$, $\mathbf{d}_e^{f'}$ and $\mathbf{E}^{f'}$ are the fluctuating dipole moment of a particle and the electric field of the blackbody radiation, and $\mathbf{d}_e^{in'}$ and $\mathbf{E}^{in'}$ are the dipole moment of a particle induced by the blackbody radiation and the electric field induced by the fluctuating dipole moment of a particle, respectively. Taking into account the statistical independence of the fluctuating values, the Lorentz force can be written in the form

$$F'_x = F'_{1x} + F'_{2x}, \quad (9)$$

where

$$F'_{1x} = \frac{\partial}{\partial x'} \langle \mathbf{d}_e^{in'} \cdot \mathbf{E}^{f'*}(\mathbf{r}') \rangle_{\mathbf{r}'=\mathbf{r}'_0}, \quad (10)$$

$$F'_{2x} = \frac{\partial}{\partial x'} \langle \mathbf{d}_e^{f'} \cdot \mathbf{E}^{in'*}(\mathbf{r}') \rangle_{\mathbf{r}'=\mathbf{r}'_0}. \quad (11)$$

To calculate F'_{1x} while writing the electric field in the K' frame as a Fourier integral

$$\mathbf{E}^{f'}(\mathbf{r}', t') = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}' \cdot \mathbf{r}' - i\omega' t'} \mathbf{E}^{f'}(\mathbf{k}', \omega'),$$

and taking into account that

$$\mathbf{d}_e^{in'} = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \int \frac{d^3k'}{(2\pi)^3} \alpha(\omega') e^{i\mathbf{k}' \cdot \mathbf{r}' - i\omega' t'} \mathbf{E}^{f'}(\mathbf{k}', \omega'),$$

where $\alpha(\omega')$ is the particle polarizability, we get

$$F'_{1x} = -i \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \int \frac{d^3k'}{(2\pi)^3} k'_x \alpha(\omega') \langle \mathbf{E}^{f'} \cdot \mathbf{E}^{f'*} \rangle_{\omega' \mathbf{k}'}. \quad (12)$$

When we change from the K' frame to the K frame $\langle \mathbf{E}' \cdot \mathbf{E}'^* \rangle_{\omega' \mathbf{k}'}$ is transformed as the energy density of a plane electromagnetic field. From the law of transformation of the energy density of a plane electromagnetic field³⁵ we get

$$\langle \mathbf{E}^{f'} \cdot \mathbf{E}^{f'*} \rangle_{\omega' \mathbf{k}'} = \langle \mathbf{E}^f \cdot \mathbf{E}^{f*} \rangle_{\omega \mathbf{k}} \left(\frac{\omega'}{\omega} \right)^2. \quad (13)$$

According to the theory of the fluctuating electromagnetic field the spectral density of the fluctuations of the electric field is determined by³⁶

$$\langle E_i^f(\mathbf{r}) E_j^{f*}(\mathbf{r}') \rangle_{\omega \mathbf{k}} = \hbar \text{Im} D_{ij}(\mathbf{k}, \omega) \coth \left(\frac{\hbar \omega}{2k_B T_1} \right), \quad (14)$$

where the Green's function of the electromagnetic field in the free space is determined by

$$D_{ik}(\omega, \mathbf{k}) = -\frac{4\pi\omega^2/c^2}{\omega^2/c^2 - k^2 + i0 \cdot \text{sgn} \omega} \left[\delta_{ik} - \frac{c^2 k_i k_k}{\omega^2} \right], \quad (15)$$

T_1 is the temperature of the blackbody radiation. Taking into account that

$$\text{Im} \frac{1}{\omega^2/c^2 - k^2 + i0 \cdot \text{sgn} \omega} = \text{Im} \frac{1}{\omega'^2/c^2 - k'^2 + i0 \cdot \text{sgn} \omega'},$$

we get

$$\langle \mathbf{E}^{f'} \cdot \mathbf{E}^{f'*} \rangle_{\omega' \mathbf{k}'} = 4\pi^2 \hbar k' \left\{ \delta \left(\frac{\omega'}{c} - k' \right) - \delta \left(\frac{\omega'}{c} + k' \right) \right\} \coth \left(\frac{\hbar \omega}{2k_B T_1} \right). \quad (16)$$

Substitution (16) in Eq. (12) and integration over ω' give

$$F'_{1x} = \frac{\hbar c}{2\pi^2} \int d^3k' k'_x \text{Im}\alpha(ck') \coth\left(\frac{\hbar\gamma(ck' + Vk'_x)}{2k_B T_1}\right), \quad (17)$$

where it was taken into account, that $\omega = (\omega' + k'_x V)\gamma$. Introducing the new variable $\omega' = ck'$, (17) can be written in the form

$$F'_{1x} = \frac{\hbar}{\pi c^2} \int_0^\infty d\omega' \omega'^2 \int_{-\omega'/c}^{\omega'/c} dk'_x k'_x \text{Im}\alpha(\omega') \coth\left(\frac{\hbar\gamma(\omega' + Vk'_x)}{2k_B T_1}\right). \quad (18)$$

At small velocities ($V \ll c$) $F_x = -\Gamma V$, where

$$\Gamma = \frac{\hbar^2}{3\pi c^5 k_B T_1} \int_0^\infty d\omega \frac{\omega^5 \text{Im}\alpha(\omega)}{\sinh^2(\frac{\hbar\omega}{2k_B T_1})}, \quad (19)$$

Equation (19) was first derived in Ref.¹¹ using a different approach. The rate of change of the rest energy of a particle in the K' frame due to the absorption of blackbody radiation is determined by the equation

$$P'_1 = \frac{dm_0}{dt'} c^2 = \langle \mathbf{j}_e^{in'} \cdot \mathbf{E}^{f'*} \rangle = \frac{\partial}{\partial t'} \langle \mathbf{d}_e^{in'}(t') \cdot \mathbf{E}^{f'*}(t'_0) \rangle_{t'=t'_0}. \quad (20)$$

After the calculations, which are similar to what was done when calculating F'_{1x} we get

$$P'_1 = \frac{\hbar}{\pi c^2} \int_0^\infty d\omega' \omega'^2 \int_{-\omega'/c}^{\omega'/c} dk'_x \omega' \text{Im}\alpha(\omega') \coth\left(\frac{\hbar\gamma(\omega' + Vk'_x)}{2k_B T_1}\right). \quad (21)$$

From Eq. (3) the friction force acting on a particle in the K -frame due to the interaction with the blackbody radiation is given by

$$\begin{aligned} F_{1x} &= F'_{1x} + \beta \frac{P'_1}{c} \\ &= \frac{\hbar}{\pi c^2} \int_0^\infty d\omega' \omega'^2 \int_{-\omega'/c}^{\omega'/c} dk'_x (k'_x + \beta \frac{\omega'}{c}) \text{Im}\alpha(\omega') \coth\left(\frac{\hbar\gamma(\omega' + Vk'_x)}{2k_B T_1}\right). \end{aligned} \quad (22)$$

Introducing the new variables: $k'_x = \gamma(q_x - \beta\omega/c)$, $\omega' = \gamma(\omega - Vk_x)$ in the integral (22) we get

$$F_{1x} = \frac{\hbar\gamma}{\pi c^2} \int_0^\infty d\omega \int_{-\omega/c}^{\omega/c} dk_x k_x (\omega - Vk_x)^2 \text{Im}\alpha[\gamma(\omega - Vk_x)] \coth\left(\frac{\hbar\omega}{2k_B T_1}\right), \quad (23)$$

where we have taken into account that $d\omega' dq'_x = d\omega dq_x$. To calculate F'_{2x} in the K' frame we use the representation of the fluctuating dipole moment of a particle as a Fourier integral

$$\mathbf{d}^f(t') = \int_{-\infty}^\infty \frac{d\omega'}{2\pi} e^{-i\omega' t'} \mathbf{d}^f(\omega'). \quad (24)$$

The electric field created in the K' frame by the fluctuating dipole moment of a particle is determined by the equation

$$E_i^{in'}(\mathbf{r}', t') = \int_{-\infty}^\infty \frac{d\omega'}{2\pi} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}' \cdot (\mathbf{r}' - \mathbf{r}'_0) - i\omega' t'} D_{ik}(\omega', \mathbf{k}') d_k^f(\omega'). \quad (25)$$

According to the fluctuation-dissipation theorem, the spectral density of the fluctuations of the fluctuating dipole moment is determined by the equation³⁶

$$\langle d_i^f d_k^{f*} \rangle_{\omega'} = \hbar \text{Im}\alpha(\omega') \coth\left(\frac{\hbar\omega'}{2k_B T_2}\right) \delta_{ik}, \quad (26)$$

where T_2 is the temperature of a particle. Substituting Eqs. (24) and (25) in Eq. (11) and taking into account Eq. (26), we get

$$F'_{2x} = -\frac{\hbar}{\pi c^2} \int_0^\infty d\omega' \omega'^2 \int_{-\omega'/c}^{\omega'/c} dk'_x k'_x \text{Im}\alpha(\omega') \coth\left(\frac{\hbar\gamma\omega'}{2k_B T_2}\right) = 0. \quad (27)$$

Thus, in the rest reference frame of a particle the friction force due to its own thermal radiation is zero. This result is due to the fact that in this frame, due to the symmetry, the total radiated momentum from the dipole radiation is identically zero. Thus the change in momentum of a particle in the rest reference frame is determined by the Lorentz force F'_x acting on a particle from the external electromagnetic field associated with the blackbody radiation observed in this reference frame. The rate of change of the rest energy of a particle in the K' frame due to its thermal radiation can be obtained with similar calculations

$$\begin{aligned} P'_2 &= \langle \mathbf{j}_e^{f'} \cdot \mathbf{E}^{in'*} \rangle = \frac{\partial}{\partial t'} \langle \mathbf{d}_e^{f'}(t') \cdot \mathbf{E}^{in'*}(t'_0) \rangle_{t'=t'_0} \\ &= -\frac{\hbar}{\pi c^2} \int_0^\infty d\omega' \omega'^2 \int_{-\omega'/c}^{\omega'/c} dk'_x \omega' \text{Im}\alpha(\omega') \coth\left(\frac{\hbar\gamma\omega'}{2k_B T_2}\right), \end{aligned} \quad (28)$$

and the friction force in the K frame associated with thermal radiation of a particle is given by

$$F_{2x} = F'_{2x} + \beta \frac{P'_2}{c} = -\frac{\hbar\gamma}{\pi c^2} \int_0^\infty d\omega \int_{-\omega/c}^{\omega/c} dk_x k_x (\omega - V k_x)^2 \text{Im}\alpha[\gamma(\omega - V k_x)] \coth\left(\frac{\hbar\gamma(\omega - V k_x)}{2k_B T_2}\right), \quad (29)$$

The total friction force in the K frame is given by

$$F_x = F_{1x} + F_{2x} = \frac{2\hbar\gamma}{\pi c^2} \int_0^\infty d\omega \int_{-\omega/c}^{\omega/c} dk_x k_x (\omega - V k_x)^2 \text{Im}\alpha(\gamma(\omega - V k_x))(n_1(\omega) - n_2(\omega')), \quad (30)$$

where $n_i(\omega) = [\exp(\hbar\omega/k_B T_i) - 1]^{-1}$. Equation (30) was first derived in Ref.¹⁹. Recently, it was derived on the basis of a relativistically covariant theory²². The approach presented in this paper is more compact and transparent, and it gives us the possibility to connect the difference between the friction forces in the different frames to the absorption and emission of radiation by a particle. Note that the friction force F_x can be either positive or negative. However, the acceleration, which is determined by the friction force F'_x , is always negative. The total heat absorbed by a particle in the K' -frame is determined by the equation

$$P' = P'_1 + P'_2 = \frac{2\hbar\gamma^2}{\pi c^2} \int_0^\infty d\omega' \int_{-\omega'/c}^{\omega'/c} dk'_x \omega' (\omega' - V q_x)^2 \text{Im}\alpha[\gamma(\omega' - V q_x)](n_1(\omega) - n_2(\omega')). \quad (31)$$

In the K frame the total change in energy of a particle due to the interaction with radiation can be calculated from the law of the transformation of energy of a particle: $E = \gamma(E' + p'_x V)$, where E and E' are the total energy of a particle in the K and K' frames, respectively. From this relation we get the equation for the rate of change of the energy of a particle in the K frame

$$\frac{dE}{dt} = P = P' + F'_x V = \frac{2\hbar\gamma}{\pi c^2} \int_0^\infty d\omega \int_{-\omega/c}^{\omega/c} dk_x \omega (\omega - V k_x)^2 \text{Im}\alpha[\gamma(\omega - V k_x)][n_1(\omega) - n_2(\omega')]. \quad (32)$$

Formula (31) was also recently obtained in²⁸ using a different approach. The rate of change of the energy of the blackbody radiation in the K frame is determined by the equation $dW_{BB}/dt = -P$. The steady-state temperature of a particle is determined by the condition $P'(T_1, T_2) = 0$, and for this state $F_x = F'_x$ and $P = F'_x V$.

III. RESULTS

The friction force acting on a particle moving relative to the blackbody radiation is determined by the imaginary part of the particle polarizability. For an atom the imaginary part of the polarizability is determined by the atom electronic linewidth broadening due to the radiation mechanism which can be calculated considering the interaction

of an atom with its own radiation. Taking into account this interaction, the dipole moment of an atom induced by an external electric field $E_x^{ext}(\omega, \mathbf{r}_0)$ can be written in the form^{18,37}

$$p_x^{ind} = \alpha_0(\omega)(\omega)[E_x^{ind}(\omega, \mathbf{r}_0) + E_x^{ext}(\omega, \mathbf{r}_0)], \quad (33)$$

where in the single-oscillator model without the radiation linewidth broadening the atomic polarizability is given by the equation

$$\alpha_0(\omega) = \frac{\alpha(0)\omega_0^2}{\omega_0^2 - \omega^2}, \quad (34)$$

$\alpha(0)$ is the static polarizability of an atom, and $E_x^{ind}(\omega, \mathbf{r}_0)$ is the radiation electric field created by the induced dipole moment of an atom. In the Coulomb gauge, which is used in this article, the Green's function of the electromagnetic field determines the electric field created by the unit point dipole, so $E_x^{ind}(\omega, \mathbf{r}_0) = \tilde{D}_{xx}(\omega, \mathbf{r}_0, \mathbf{r}_0)p_x^{ind}$, where $\tilde{D}_{xx}(\omega, \mathbf{r}_0, \mathbf{r}_0)$ is the reduced part of the Green's function of the electromagnetic field in the vacuum, which takes into account only the contribution from the propagating electromagnetic waves and determines the radiation in the far field. The Green's function of the electromagnetic field in the vacuum $D_{xx}(\omega, \mathbf{r}_0, \mathbf{r}_0)$ diverges at $\mathbf{r} = \mathbf{r}_0$. However, the contribution from the propagating waves remains finite and purely imaginary at $\mathbf{r} = \mathbf{r}_0$, and the divergent contribution from the evanescent waves is real. Therefore, $\tilde{D}_{xx}(\omega, \mathbf{r}_0, \mathbf{r}_0) = i\text{Im}D_{xx}(\mathbf{r}_0, \mathbf{r}_0)$. From Eqs. (33) and (34) we get

$$\text{Im}\alpha(\omega) = \text{Im}\frac{p_x^{ind}}{E_x^{ext}(\omega, \mathbf{r}_0)} = \text{Im}\frac{\alpha(0)\omega_0^2}{\omega_0^2 - \omega^2 - i\alpha(0)\omega_0^2\text{Im}D_{xx}(\mathbf{r}_0, \mathbf{r}_0)} = \frac{\alpha^2(0)\omega_0^4\text{Im}D_{xx}(\mathbf{r}_0, \mathbf{r}_0)}{(\omega_0^2 - \omega^2)^2 + [\alpha(0)\omega_0^2\text{Im}D_{xx}]^2}, \quad (35)$$

where

$$\text{Im}D_{xx}(\mathbf{r}_0, \mathbf{r}_0) = \text{Im}D_{yy} = \text{Im}D_{zz} = \int \frac{d^3k}{(2\pi)^3} \text{Im}D_{xx}(\omega, \mathbf{k}) = \frac{2}{3} \left(\frac{\omega}{c}\right)^3 \text{sgn}\omega. \quad (36)$$

At the resonance ($\omega^2 \approx \omega_0^2$) usually $\alpha(0)\text{Im}D_{xx} \ll 1$ (for example, for a hydrogen atom it is $\sim 10^{-6}$) thus the limit $\alpha(0)D_{xx} \rightarrow i0$ can be taken. In this case the resonant contribution is given by

$$\text{Im}\alpha(\omega) \approx \frac{\pi\alpha(0)\omega_0}{2} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)], \quad (37)$$

and the off-resonant contribution in the field far from resonance ($\omega^2 \ll \omega_0^2$)

$$\text{Im}\alpha(\omega) \approx \frac{2}{3} \left(\frac{\omega}{c}\right)^3 \alpha^2(0) \text{sgn}\omega. \quad (38)$$

Recently, the result (38) was also obtained using quantum electrodynamics³⁰. However, the analysis presented in this paper, is much simpler, and it clarifies the physical meaning of the terms of the series of the perturbation theory of quantum electrodynamics. Using Eqs. (37) and (38) in Eq. (18) we get the resonant and off-resonant contributions to the friction force

$$F_{1x}^{res} = \frac{\hbar\omega_0^5\alpha(0)}{c^4} \int_{-1}^1 dx \left[\exp\left(\frac{\hbar\gamma\omega_0(1+\beta x)}{k_B T_1}\right) - 1 \right]^{-1}, \quad (39)$$

$$F_{1x}^{nonres} = -\frac{512\pi^7\hbar\alpha(0)^2\gamma^6}{945c^7} \left(\frac{k_B T_1}{\hbar}\right)^8 (7\beta + 14\beta^3 + 3\beta^5), \quad (40)$$

At $\beta \ll 1$ the friction force $F_{1x} = -\Gamma V$, where the resonant and off-resonant contributions to the friction coefficient are

$$\Gamma_{res} = \frac{\hbar^2\alpha(0)\omega_0^6}{6c^5k_B T_1} \frac{1}{\sinh^2\left(\frac{\hbar\omega_0}{2k_B T_1}\right)}, \quad (41)$$

$$\Gamma_{nonres} = \frac{512\pi^7\hbar\alpha(0)^2}{135c^8} \left(\frac{k_B T_1}{\hbar}\right)^8, \quad (42)$$

Results (41) and (42) were already obtained in Refs.^{11,30}. In the ultrarelativistic case ($1 - \beta \ll k_B T_1 / \hbar \omega_0 \ll 1$)

$$F_{1x}^{res} = \frac{\omega_0^4 \alpha(0)}{c^4} \sqrt{2} k_B T_1 \sqrt{1 - \beta} \ln \frac{\hbar \omega_0 \sqrt{1 - \beta}}{\sqrt{2} k_B T_1}, \quad (43)$$

$$F_{1x}^{nonres} = \frac{216 \pi^7 \hbar \alpha(0)^2}{135 c^7 (1 - \beta)^3} \left(\frac{k_B T_1}{\hbar} \right)^8 \quad (44)$$

Thus in the ultrarelativistic case $F_{1x}^{res} \sim \sqrt{1 - \beta} \ln \sqrt{1 - \beta} \rightarrow 0$ and $F_{1x}^{nonres} \sim (1 - \beta)^{-3} \rightarrow \infty$ at $1 - \beta \rightarrow 0$.

For small velocities and typical temperatures the infrared thermal peak of the blackbody radiation is far below the resonance frequency ω_0 , so it dominates the far-off-resonant contribution (41), which has already been mentioned in Ref.²⁹. However, in the ultrarelativistic case the friction is dominated by the far-off-resonant contribution for all temperatures.

According to Eq. (31) the total heat absorbed by an atom in the K' frame is determined by the equation

$$P' = \frac{128 \pi^7 k_B^8 \alpha(0)^2}{315 c^6 \hbar^7} [\gamma^6 (7 + 35\beta^2 + 21\beta^4 + \beta^6) T_1^8 - 7 T_2^8]. \quad (45)$$

For small velocities the steady-state temperature of a particle $T_2 = T_1$ and in the ultrarelativistic case $T_2 \approx (1 - \beta)^{-3/8} T_1$. The problem in calculating the imaginary part of the atom's polarizability in the far off-resonant field was considered in Ref.²⁹. It was noted that in the literature there are still questions still remain regarding the gauge invariance of the imaginary part of the polarizability. In this paper, the imaginary part of the atomic polarizability in the field far from resonance is determined by the imaginary part of the electric field of the unit point dipole, which is a gauge-invariant quantity. In the Coulomb gauge, which is used in this article, the electric field of the unit point dipole is the same as the Green's function of the electromagnetic field. At another gauge the expression for the Green's function will changed; however, the electric field determined with this Green's function will remain unchanged. Therefore, despite the fact that the Green's function for the electromagnetic field is a gauge-dependent quantity, the imaginary part of the atomic polarizability calculated in this article is a gauge-invariant quantity. The gauge invariance of the obtained results are also confirmed by the direct calculation using quantum electrodynamics³⁰. The gauge-invariant formulation presented in this paper confirms that the polarizability of the atom, for small frequencies, is a nonresonant effect, which is proportional to ω^3 for small driving frequency ω . This is consistent with the gauge-invariant analysis conducted in Ref.³⁰.

IV. DISCUSSION

The theory of the friction force at the relativistic movement of a small neutral particle relative to the blackbody radiation was developed by Dedkov and Kyasov^{19,27,28} (DK) in a series of works in which the friction force in the K frame was calculated. Recently DK's results were confirmed within the framework of a fully covariant theory²². In Ref.¹⁶ we proposed an alternative theory (VP) in which the friction force in the K' frame was calculated. This theory was criticized by DK in Ref.²⁷. In this paper, a general theory which includes previous theories and establishes a link between them is developed. Within this theory we refute the criticisms of the VP theory made by DK. The key point in the VP theory is the establishment of the relation between the friction forces in the K and K' frames. According to the VP theory these friction forces are generally not equal but differ due to the change in the rest mass of a particle as a result of absorption and emission of radiation by a particle. This result is in sharp contradiction to the opinion of DK in Ref.²⁷ which overlooked the effect of changing the rest mass of a particle. As a result, they came to the conclusion (which we find to be incorrect) that in the general case the total friction forces, which consist from the contributions from the interaction of a particle with the blackbody radiation and with its own radiation, in the K and K' frames are equal. DK suggested²⁷ that, in the general case, the contribution to the friction force from the interaction of a particle with its own radiation in the K' frame is not equal to zero, which we believe is incorrect. In the present paper the result that this frictional force is equal to zero is proved by rigorous calculation. However, this result is evident from the symmetry of the dipole radiation in the K' frame. Due to this symmetry the momentum emitted by a particle, and hence the force due to the interaction of a particle with its own radiation, is equal to zero. So, if we follow the reasoning of DK, the total friction force in the K frame in general case should be equal to the contribution to the friction force in the K' frame due to the interaction of a particle with the blackbody radiation, which we find to be incorrect and contradicts opinions stated by DK. As shown in the present work, the equality of the total friction forces in the K and K' frames takes place only in the special (but physically most important) case of motion of a particle with a steady-state temperature. In this case the particle rest mass does not change and friction

forces in the K and K' frames are equal to the friction force in the K' frame due to the interaction of a particle with the blackbody radiation, which also contradicts the opinion of DK. In addition, considering the relativistic dynamics DK stated that the acceleration of the particle is determined by the friction force in the K frame, which can be positive or negative. However, according to the VP theory the particle acceleration is determined by the friction force in the K' frame, which is always negative.

V. CONCLUSION

It was shown that in the most simple way the friction force on a neutral particle at the relativistic motion relative to the blackbody radiation can be calculated in the rest frame of a particle (K' frame) where the friction force is determined only by the interaction of a particle with the blackbody radiation. In the frame of the blackbody radiation (K frame) the friction force can be found by using the Lorentz transformation for the friction force. The friction force in the K frame is determined by both the interactions of a particle with the blackbody radiation and the thermal radiation of a particle and can be either positive or negative. However, the acceleration, which is determined by the friction force in the K' frame, is always negative. The difference between the forces in the different frames is connected to a change in the rest mass of a particle due to the absorption and emission of radiation by a particle. For the steady-state temperature of a particle the friction forces in the K' and K frames are equal. For an atom, blackbody radiation friction is determined by the radiation electronic linewidth broadening, which can be calculated by considering the interaction of an atom with its own radiation. The obtained results can be used to studying the interaction of cosmic rays with background blackbody radiation and noncontact friction. For example, the approach proposed in this paper for calculating the radiation friction can also be used to calculate quantum Vavilov-Cherenkov radiation³⁸ by a neutral particle moving parallel to a dielectric.

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